A Rayleigh link to overcome fog attenuation

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ABSTRACT
In case of fog ground to ground free space link with a laser is almost impossible because of the strong attenuation. In principle this event is relatively rare and the usual solution is to provide an alternate radio link in such a situation. However, in several practical cases, optical links for the 'last mile' are required in order to avoid to obtain licensing for a radio channel from local authorities. Usually, however, the fog has a vertical thickness that is very limited. It is not uncommon, for instance, that in foggy nights one can easily see 'stars' while a direct vision over the ground is completely forbidden. In this case one can send a modulated beam 'onto the sky' and recover it from its side by means of Rayleigh scattering. In order to achieve competitive data rate, however, one is forced to make on - ship fast tracking of the pulse train, in order to accumulate signal for a significant range of the Rayleigh beacon. A preliminary photon budget shows that such an approach can give interesting data rate with reasonable laser power and affordable optics. A preliminary discussion of potential drawbacks and circumventing possibilities is also reported.

1. INTRODUCTION
The Astronomical Observatory of Padova currently holds, as many Astronomical Observatories located in relatively big cities in the world, only old telescopes for historical and museal purposes rather than for direct astronomical research. Real astronomical observatories are located in the Alps and in remote places, like our 1.8m telescope in Asiago and our 3.5m telescope located in Canary Island. That is not surprising. Even without thinking about seeing or other relatively novel concept items, especially during the winter season, the Observatory, located in the Po valley, is often plagued by fog. It is nice to realize, however, that in a number of occasions fog can be so thick to place some additional danger in car - driving, but you can still give a look to near zenith constellations by eye. This is a relative common experience in this area. Moreover in some occasions, landing by airplane in the near Venice airport one can see fog as a sort of white slab smoothing out street - lights. Although all these are very crude and subjective observations the main guideline behind these is that fog is confined to a limit thickness over the ground.

As Astronomical Observatory we are pursuing a technology development program for Adaptive Optics. In this framework we study laser propagation in the free atmosphere and, especially, we outlined a few techniques implementing observations and sensing of Rayleigh beacons\textsuperscript{1,2}. We simply applied a few of these concepts to propose a new approach for optics communication under such extreme foggy situation.

2. A RAYLEIGH BRIDGE TO JUMP ACROSS THE FOG
The finite height of the fog slab suggested to use Rayleigh scattering in the free atmosphere as a sort of natural transponder. Here we are proposing to fire a laser beam at a certain elevation. In this way of course the laser light will never reach directly the receiving station. However the attenuation during the path in the fog will be by far less important (of course as much as the distance between the two station is larger than the thickness of the fog slab). Scattering of light in a Rayleigh fashion is of course a very inefficient way to redirect some light toward the receiver station. However one can use some tricks in order to achieve enough Signal to Noise Ratio. As it is depicted in Fig.1 a pulse train is injected into the atmosphere and it can be seen by side,
Figure 1. A train of pulses is launched onto the sky and, by Rayleigh scattering, is observed from a side telescope. The speed of the train beacon is just slightly smaller than \(c\), but the speed on the focal plane of the receiving telescope is diminished by the optical lever. The ratio can be as large as 4 orders of magnitude and the CCD is employed with a fast charge shifting during the exposure, so that a single pulse can accumulate enough Signal to Noise Ratio while it goes through the covered Field of View.

because of Rayleigh scattering, from a receiving telescope. The speed of the train pulse is \(\approx c\) and we assume that the beacon is reimaged onto a linear CCD. Let us assume a distance from the beacon to the receiving station of the order of \(\approx 2\text{km}\). Since a linear CCD array of, let say, \(10^3\) pixel \(\approx 15\mu\text{m}\) in size, is optically matched to the column beacon with a Field of View of half a radian, one is matching a column of 500m length to a 15mm linear length on the focal plane. The speed of a pulse is so hampered by the same linear ratio between the two lengths considered here, that is by a factor \(\approx 3.3 \times 10^4\). Speed of the pulse translates on the focal plane as a mere \(9.1 \times 10^3\text{m}\text{s}^{-1}\). This speed, expressed in pixels per second, rather than in meters per seconds, gives the clock frequency one need to drive the CCD in order to accumulate on the same \textit{logical} pixel all the light scattered by the pulse during its journey along the beacon column. Such a figure turns out to be, in the above mentioned example, as \(\approx 364\text{MHz}\) a value that is of the same order of what is currently available in the market for this type of detector. Of course in this case focal length should be kept so small that one is questioned by the focal ratio of the receiving station, in order to increase the aperture diameter. We do not go into any detail here, however we just point out that one need such a plate scale only along a specific direction so that an anamorphic relay with a rectangular aperture can be used, in order to achieve enough aperture without need for particularly fast optical design.
Figure 2. Basic geometrical data for the setup considered in the text. We assume the fog is a slab of thickness \( h \) with constant characteristic attenuation length \( k_0 \). Also, being \( d \) usually of the order of one mile, we assume atmospheric pressure constant in the volume where the Rayleigh beacon is deployed and observed.

3. PHOTON BUDGET

Let us assume that two optical stations at a distance \( d \) experience a severe fog situation (see Fig.2). Here fog is characterized by two figures: the height \( h \) of the fog slab and the characteristic attenuation length \( k_0 \). The latter is such that light intensity \( I_0 \) covering a pathlength \( d \) along an horizontal path will experience an attenuation given by:

\[
I_h = I_0 \exp \left( -\frac{h}{l_0} \right)
\]

usually in the technical literature attenuation is expressed in dB/km and figures in the range 30...300dB/km are experienced. Recalling that 20dB corresponds to a factor 10 in attenuation it is easy to see that the reported figures corresponds roughly to \( l_0 \sim 30...300 \)m (see for instance Ref.3).

Let us suppose, in this case, that the projector is tilted of 45 degree upward and that the receiving station will collect Rayleigh scattered photons along a certain Field of View. One can use the so-called Lidar equation\(^4\) to retrieve the amount of collected photons. The scattered photons by a laser pulse of energy \( E \) fired at a range \( r \), where the average pressure is \( P \), gives back on the ground a number of photons per square meter and per meter of Rayleigh column, given by:

\[
N_\gamma (\text{photons} \times m^{-3}) \approx 6.7 \times 10^{10} \frac{E(\text{Joule}) \cdot P(\text{mBar})}{h^2(\text{m}) \cdot \lambda^4(\mu m)}
\]

In case of the Rayleigh setup shown in Fig.2 one can say that the efficiency of transfer of photons from the transmitter to a receiver of aperture \( D \) is given by the ratio of the collected scattered photons by the emitted ones. Recalling that the measurement is obtained by integrating along a Rayleigh column of size \( \theta d / \sqrt{2} \) one can easily find:

\[
\eta \approx \frac{h c}{E \lambda} \frac{\theta d}{\sqrt{2}} \times N_\gamma \left[ P \left( \frac{d}{\sqrt{2}} \right) ; h = \frac{d}{\sqrt{2}} \right]
\]
Working out the latter relationship one get the following:

\[ \eta \approx 1.5 \times 10^{-14} \theta D^2 P(\text{mBar}) / d X (\mu m) \]  

(4)

A few comments are necessary to devise out the physical meaning of the various power-law dependencies:

1. The fifth power law for wavelength is a combination of Rayleigh scattering law (a fourth power law) and the assumption to fix up the energy of the laser rather than the overall number of photons. A strong dependence upon the wavelength indicates that the Rayleigh approach described here is strongly weighted toward the shortest wavelength usable.

2. Dependence upon station's distance is only inversely linear. In fact, once distance is augmented, the collected portion of the Rayleigh beam increases, because we fixed up a certain FoV \( \theta \) for the receiver. This partially compensates the inverse square law because of the distance of the Rayleigh scattered light from the beacon to the receiver. Note that all the beacon is imaged, hence there is no any attenuation effect because of the distance from the laser source to the beacon region.

3. The dependence on the collecting surface assumes that in the horizontal case all of the energy is collected by the receiving station and that no loss in power is experienced at this level, so the estimate given here is a conservative limit.

As a case study we can take a FoV \( \theta = 0.5 \) radians (around 28.5°), an aperture of \( D = 0.4m \), a distance \( d = 2 \times 10^3 \)m and a corresponding average pressure \( P \approx 1000\,\text{mBar} \), and a wavelength of \( \lambda = 0.3 \mu \text{m} \). An efficiency of \( \eta \approx 2.4 \times 10^{-13} \) is obtained. Although this number sounds very low one should compare it with log attenuation that amount to \( \exp(-d/l_0) \approx 1.1 \times 10^{-20} \) whenever a \( l_0 = 30 \text{m} \) is used. In the configuration used in this example, hence, the Rayleigh scattering approach is around 16 orders of magnitude more efficient when the approximation of \( h \ll l_0 \) is used. Generally speaking the comparison between the two techniques can be given by estimating the ratios between the transmitted energy as:

\[ \frac{I_r}{I_h} \approx \eta \exp \left( \frac{d - 2\sqrt{2}h}{l_0} \right) \]  

(5)

With a figure of \( h = 100\text{m} \) the relative gain drops from \( \approx 10^{16} \) to only 15 orders of magnitude.

Provided that the efficiency is much larger one should, by the way, consider which are the types of power that can be adopted to transmit at a reasonable bandwidth. Let us assume that at SNR=3 one makes a single bit detection, and, because of monochromaticity, we also assume negligible background noise sources and an overall instrumental efficiency of \( \rho = 0.1 \). Under these conditions it is easy to see that one needs \( \approx 100 \) photons to achieve a single bit detection. To achieve a certain bandwidth \( B \) in bps one needs a power \( W \) on the emitting laser given by:

\[ W \approx 1.3 \times 10^{-9} \frac{Bd\lambda^4}{\theta \rho D^2 P(\text{mBar})} \exp \left( \frac{2\sqrt{2}h}{l_0} \right) \]  

(6)

Under the conditions we have outlined we obtain \( W \approx 3.25 \times 10^{-5}B \). For instance, there is need of \( W \approx 3.25 \text{Watt} \) to transmit 100Kbps, while, on the other hand, a more reasonable \( W = 0.5 \text{Watt} \) leads to an achievable data rate of \( B \approx 15.4 \text{Kbps} \). Of course we always refer to rough, uncompressed data transmission.

4. CONCLUSIONS

Our mission is not intended to build up commercially usable optics link system. We are aware that the described approach has to be verified from the commercial point of view in order to establish the size of its potential market, if any, and its cost–effectiveness. We think, however, we have injected in the community a somehow new approach that could, in this or modified form, spread new ideas in the optics communication field. It is in this spirit that we performed the analysis described in the text and it is in this spirit that the same should be read.
### Table 1

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Value and unit</th>
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<tbody>
<tr>
<td>Distance between stations</td>
<td>$d$</td>
<td>2000m</td>
</tr>
<tr>
<td>Laser wavelength</td>
<td>$\lambda$</td>
<td>0.3\mu m</td>
</tr>
<tr>
<td>Receiver FoV</td>
<td>$\theta$</td>
<td>0.5 radians $\approx 28.6^\circ$</td>
</tr>
<tr>
<td>Air pressure</td>
<td>$P$</td>
<td>1000 mBar</td>
</tr>
<tr>
<td>Receiver aperture diameter</td>
<td>$D$</td>
<td>0.4m</td>
</tr>
<tr>
<td>Detection efficiency</td>
<td>$\rho$</td>
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<tr>
<td>Fog thickness</td>
<td>$h$</td>
<td>100m</td>
</tr>
<tr>
<td>Attenuation distance</td>
<td>$l_0$</td>
<td>30m</td>
</tr>
</tbody>
</table>

We report here all the parameters we used for the example discussed in the text.

**REFERENCES**